HYDRODYNAMIC MODE IN A THREE-PHASE FIXED GRANULAR LAYER. THEORETICAL ANALYSIS

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A theoretical analysis of the hydrodynamic modes in a fixed granular layer is carried out for an ascending direct flow of gas and liquid. A number of hydrodynamic models are proposed and conditions for the passage from one mode to another are determined.

The flow conditions for a gas—liquid mixture in the space between grains of a packed bed (PBF) govern heat, mass, and momentum transport to a considerable degree. Knowing them becomes especially necessary in the development of a number of highly intensive technological processes in energetics, petrochemistry, and chemical technology.

The modern state of the art of the theory of disperse media flow does not afford the possibility of describing the origin and existence of different hydrodynamic modes in a packed bed. Hence, in our opinion, the examination of a number of physical models of gas—liquid mixture motion in a granular bed and the determination of critical conditions for the passage from one mode to another are most convenient.

To clarify the regularities of gas and liquid motion in the free volume of the layer, qualitative experimental investigations were conducted in a transparent 100-mm-diameter column with an ascending direct flow of gas and liquid. It was established that depending on the discharges and dimensions of the packing in the PBF, five of the most characteristic hydrodynamic modes could be isolated: 1) bubble; 2) shell in the channels between grains; 3) piston in the scale of the whole bed; 4) annular-disperse; 5) drop.

The bubble mode is characterized by the fact that the gas motion is accomplished in the form of bubbles insulated from each other. The size of the bubble depends on the distributive units and geometric structure of the packed bed, and the rate of ascension is determined by the balance between the mechanical forces acting on the ascending bubble.

If the spacing between elements of the bed is greater than the bubble size determined by the distributive unit, then the gas motion is accomplished within the scales of one grain. In the opposite case, a bubble encompasses several grains during its motion through the granular layer. It can be assumed that an inequality following from the Taylor theory of instability [1] is the boundary of these domains:

$$R^{cr} \gtrsim \sqrt{\frac{\sigma}{g(\rho_2 - \rho_1)}}\,.$$

A theoretical analysis and the experimental results presented in [2] show that the bubble structure in a bubbling layer can exist up to a 0.3-0.4 gas content.

Spoilage of the flow mode and the passage to a shell mode are observed for large values of the gas fraction. The critical conditions of this passage can be determined for a packed bed if conditions for the approach and merger of successive bubbles are known. It is hence necessary to take their mutual influence into account. Thus, if the spacing between bubbles is less than 2-3 diameters, then, as follows from [3], the second bubble is incident in the wake of the leader (first bubble) and its velocity will increase as they approach. For large spacings the bubble motion occurs independently.

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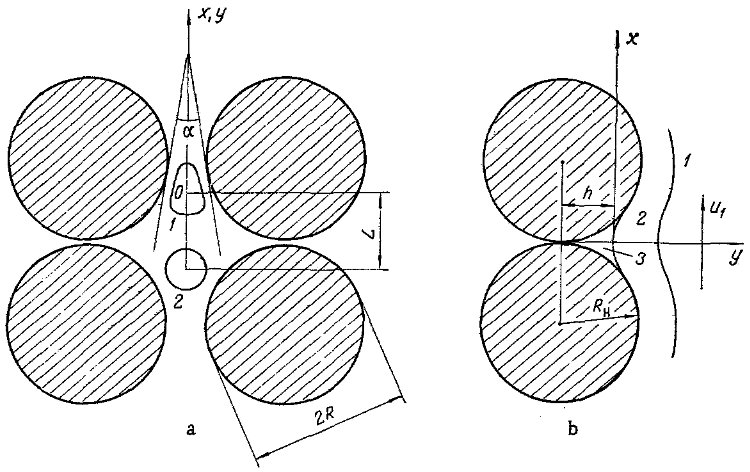

Fig. 1. Geometric models of bed sections: a) model of bubble structure (1 is the leading bubble and 2 is a bubble in the wake of the first); b) annular flow model (1 is the gas core of the stream, 2 is a liquid film, and 3 is a fixed fluid).

If the space between the grains is represented as a channel of variable cross section, and the bubble therein by a spherical sector, then the mathematical description of the motion of the centers of gravity for a pair of approaching bubbles has the form

$$A_1 - A_2 \frac{dx}{d\tau} - A_3 x = \mu \frac{d^2x}{d\tau^2}, \tag{1}$$

$$B_1 - B_2 \frac{dy}{d\tau} + \mu \frac{dW}{d\tau} = \mu \frac{d^2y}{d\tau^2}, \tag{2}$$

where

$$A_1 = B_1 = \frac{g(\rho_2 - \rho_1) R_0}{\rho_2 u_0^2}; \quad B_2 = \frac{12\pi\mu_2 R_0^2}{\rho_2 u_0 V} \frac{1 - \varepsilon^{5/3}}{(1-\varepsilon)^2};$$

$$A_2 = B_2 \sqrt[3]{\frac{2}{1 - \cos\alpha/2}}; \quad A_3 = \frac{2\pi\sigma f(\alpha) R_0^2}{\rho_2 u_0^2 V}; \quad \mu = \frac{\rho_1}{\rho_2};$$

$$\tau = \frac{u_0 t}{R_0}; \quad x = \frac{x_1}{R_0}; \quad y = \frac{y_1}{R_0};$$

$$f(\alpha) = \frac{\sin\frac{\alpha}{2} + 2\left(1 - \cos\frac{\alpha}{2}\right)}{\frac{3}{4} + \frac{5}{8}\left(1 - \cos\frac{\alpha}{2}\right)};$$

for $\tau = 0$: $x = 0$, $y = -L/R_0$, $dx/d\tau = dy/d\tau = 1$.

Equations (1) and (2) take account of the influence on the bubble motion of gravity, friction, and deformation in connection with narrowing of the channel [4], as well as acceleration of the second bubble caused by motion of the first. According to [3], this last acceleration can be represented as

$$W = \frac{dx}{d\tau}\left[8\exp\left(-\frac{R_0(x-y)}{D}\right)\right]. \tag{3}$$

Since the coefficient of the highest derivative in (1) and (2) is a small quantity, then an asymptotic expansion of the solutions can be written on the basis of [5]. Thus, by manipulating the system presented above, we obtain

$$\frac{dx}{d\tau} = u, \tag{4}$$

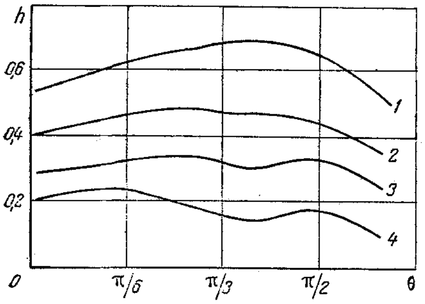

Fig. 2. Dependence of the height of the capillary rise on the packing radius and the wettability angle: 1) R = 1 cm; 2) 0.75; 3) 0.15; 4) 0.25 h, cm.

$$\mu \frac{du}{d\tau} = A_1 - A_2 u - A_3 x,$$
$$\frac{dy}{d\tau} = Y, \tag{5}$$
$$\mu \frac{dY}{d\tau} = B_1 - B_2 Y + \mu \frac{dW}{d\tau}.$$

Omitting all the intermediate computations because of their awkwardness, let us just present the expressions for the first approximations:

$$x = \frac{A_1}{A_3}\left[1 - \exp\left(-\frac{A_3}{A_2}\tau\right)\right] + \frac{\mu}{A_2}\left[\left(\frac{A_1 A_3}{A_2^2}\tau - 1 + \frac{A_1}{A_2}\right)\right.$$
$$\left.\times \exp\left(-\frac{A_3}{A_2}\tau\right) + 2\left(1 - \frac{A_1}{A_2}\right) - \left(1 - \frac{A_1}{A_2}\right)\exp(-A_2\theta)\right], \tag{6}$$

$$u = \frac{A_1}{A_2}\exp\left(-\frac{A_3}{A_2}\tau\right) + \left(1 - \frac{A_1}{A_2}\right)\exp(-A_2\theta)$$
$$+ \mu\frac{A_3}{A_2^2}\left[\left(1 - \frac{A_1 A_3}{A_2^2}\tau\right)\exp\left(-\frac{A_3}{A_2}\tau\right) + 2\left(1 - \frac{A_1}{A_2}\right)\right.$$
$$\left. - \left(1 - \frac{A_1}{A_2}\right)\theta\exp(-A_2\theta)\right], \tag{7}$$

$$y = -\frac{L}{R_0} + \frac{B_1}{B_2}\tau + \frac{\mu}{B_2}\left\{\int f_1(\tau)\,d\tau - \int f_1(\tau)\,d\tau\Big|_0\right.$$
$$\left. + \left(1 - \frac{B_1}{B_2}\right)[1 - \exp(-B_2\theta)]\right\}, \tag{8}$$

$$Y = \frac{B_1}{B_2} + \left(1 - \frac{B_1}{B_2}\right)\exp(-B_2\theta) + \left\{\frac{f_1(\tau)}{B_2} + \frac{8R_0}{B_2 D}\left(1 - \frac{B_1}{B_2}\right)\right.$$
$$\left.\times \exp\left(\frac{R_0 L}{D^2} - B_2\theta\right) - \exp(-B_2\theta)\left[\int f_2(\tau)\,d\theta\Big|_0 - \int \exp(B_2\theta) f_2(\tau)\,d\theta\right]\right\}, \tag{9}$$

$$f_1(\tau) = 8\left[\frac{d^2x}{d\tau^2} - \frac{R_0}{D}\left(\frac{dx}{d\tau}\right)^2 + \frac{R_0}{D}\frac{B_1}{B_2}\frac{dx}{d\tau}\right]$$
$$\times \exp\left[-\frac{R_0}{D}\left(x + \frac{L}{D} - \frac{B_1}{B_2}\tau\right)\right],$$

$$f_2(\tau) = 8\left[\frac{d^2x}{d\tau^2} - \frac{R_0}{D}\left(\frac{dx}{d\tau}\right)^2\right]\exp\left(-\frac{R_0 x}{D}\right),$$

$$\theta = \frac{\tau}{\mu},\ L = \frac{1-\varphi}{\varepsilon\varphi}\Delta L.$$

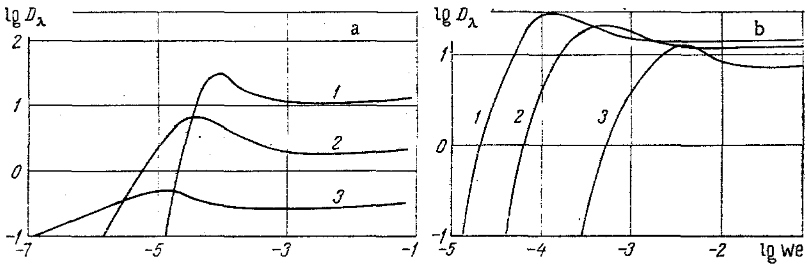

Fig. 3. Influence of the numbers We and δ/h [1) δ/h = 0.1; 2) 0.5; 3) 1.0] (a) and We and γ [1) γ = 0; 2) 0.66; 3) 1.0] (b) on the dimensionless wave number.

As follows from the geometric model taken (Fig. 1a), the approach of the bubbles is possible at spacings equal to the grain radius R. For uniform motion of the bubbles at the velocity u_0, the spacing between them x—y will remain constant and equal to L/R_0, while $\tau = (x-y)/u_0 = R/R_0$. Therefore, if the time to traverse this spacing under the nonuniform motion determined from the solution of (6)-(9) is less than R/R_0, then the bubbles approach; otherwise, they diverge.

An analysis of (6)-(10) shows that we can limit ourselves to the zero approximation in computing the velocity of bubble motion and the spacing between them. The error associated with neglecting terms containing μ is not more than 5%. The condition for passage from the bubble mode to the emulsion foam or shell will have the following form in this case:

$$1 > \frac{1-\varphi}{\varepsilon\varphi} - \frac{B_1}{B_2} + \frac{R_0}{R}\frac{A_1}{A_3}\left[1-\exp\left(-\frac{A_3 R}{A_2 R_0}\right)\right]. \tag{10}$$

The approaching bubbles can move in the form of nonassociated agglomerates or shells depending on the difference between the dynamic pressures acting on the phase interfaces and the viscous forces stabilizing the film. Thus, if

$$\frac{\rho_2 (u_1^2 - u_2^2)}{2} > \mu_2 \frac{u_1 - u_2}{\Delta}, \tag{11}$$

then a discontinuity occurs in the surface of separation and the bubbles unite into a shell; otherwise, an emulsion motion mode is observed. The quantity Δ characterizes the film thickness at which the discontinuity starts and this can be determined experimentally.

The shell mode in the packing channels is characterized by the motion of relatively stable large gas bubbles whose length is considerably greater than their diameter and comprises several packing dimensions. An increase in the gas discharge will result in a growth of the shell length and a diminution in the thickness of the liquid connectors. In the long run this results in merging of the shells and the formation of channels in the space between the grains. The conditions for the passage to such a mode can be obtained analogously to [6] in the form

$$\bar{m}(u_1 + u_2 + k\sqrt{gD}) = u_1, \tag{12}$$

where $\bar{m}$ is an empirical constant, equal to 0.943 for a laminar flow, and k = 0.35 [6].

Let us consider the singularities of fluid motion through a granular bed in the case of long shell or gas-channel formation. The hydrodynamic model is presented in Fig. 1b. The mathematical description for perturbed flow can be represented as [7-9]

$$\frac{\partial p}{\partial x} = -\rho_2\left(\frac{\partial u'}{\partial t} + v\frac{\partial u'}{\partial x} + v\frac{dv'}{dy}\right) \tag{13}$$

in conformity with the linear theory of stability for zone 2. Later, going over to the stream functions

$$\psi = \varphi(y)\exp[i(mx - \omega t)]u' = \frac{\partial\psi}{\partial y},\ v' = \frac{\partial\psi}{\partial x}, \tag{14}$$

let us write the boundary conditions for the corresponding zones:

$$y = -\delta : \varphi_1 = \varphi_2,\ \frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x} = -\sigma\frac{\partial^3\eta}{\partial x^3} \tag{15}$$

This condition characterizes the balance of the forces on the interphase surface of separation [8] taking surface tension into account:

$$y=0:\varphi_2=\varphi_3,\quad \frac{\partial p_2}{\partial x}=\frac{\partial p_3}{\partial x};$$

$$y=h:\ \frac{\partial\varphi}{\partial y}=0,$$

where h is the height to which the fluid rises in the variable-section capillary. It can be determined from the solution of the transcendental equation

$$\rho_2 gh=\frac{\cos\theta+r'\sin\theta}{\sqrt{1+(r')^2}}\,\frac{2\sigma}{r}, \tag{16}$$

where $r(h)=R-\sqrt{R^2-h^2}$; $r'=dr/dh$, numerical results of which are presented in Fig. 2.

The analysis of the system (13)-(16) reduces to determining the dependence of the vibration increment on the wave number for different values of the parameters We, δ/h, γ. The results, obtained on a "Minsk-32" electronic digital computer, are represented in Fig. 3, where values of the dimensionless wave number corresponding to the maximum of the vibration wave number are plotted along the ordinate axis. The curves presented determine the boundary between the stable and unstable domains.

It is seen from this figure that for high values of δ/h the fluid film is unstable in practically the whole range of variation of We. This means that motion of the gas and liquid in separate channels is more stable in a granular layer consisting of small-size packing elements. In the terminology of Fig. 3, this corresponds to an effective diminution in the quantity δ/h and an increase in D_λ, which results in stability. The channel size which is hence obtained can be estimated by means of the formula

$$R^{cr}\approx\frac{\lambda}{2}\approx\frac{\pi h}{D_\lambda}. \tag{17}$$

Therefore, the problem of destruction of a gas channel for packings in which the dimension is less than R^{cr} determined from (17) reduces to the problem of dissociation of a jet in an unbounded medium. An analysis carried out in [7] has shown that this phenomenon is observed for wave-number values equal to $^2/_3$ We. The instability condition for the phase separation boundary, which follows from the theory of dynamic waves [8], hence has the form

$$2<\frac{u_1(1-\varphi)}{u_2\varphi}+\frac{u_2\varphi}{u_1(1-\varphi)}-\frac{\sigma m\varphi(1-\varphi)}{u_1u_2}\cdot\frac{\rho_2-\rho_1\operatorname{cth}\frac{2}{3}\mathrm{We}}{\rho_1\rho_2\operatorname{cth}\frac{2}{3}\mathrm{We}}. \tag{18}$$

If the granular layer consists of elements whose linear dimension is greater than R^{cr}, then the motion becomes unstable only for definite values of the numbers We and γ (Fig. 3). The channel size which can be obtained analogously to (17) is less than the geometric size in this case, and it is hence natural to assume that separation of the fluid drops from the surface of the moving film occurs rather than the formation of new channels; i. e., the passage over to the drop mode is observed. Therefore, for such packings (12) determines the passage over to a disperse-annular mode.

The condition for going over to the drop mode can be obtained from the equation of wave propagation of the phase separation surface, which can be obtained in case of the presence of a solid wall and free boundaries according to Milne-Thompson [8]:

$$\rho_2(u_2-c)^2\operatorname{cth}m(a-2\delta)+\rho_1(u_1-c)^2\operatorname{cth}m(h+\delta)=\sigma m. \tag{19}$$

If (17) has no real roots in the dynamic velocity c, then the surface of separation is unstable, and the condition for fluid film destruction (this condition simultaneously governs the passage from the disperse-annular to the drop mode) appears as follows:

$$(u_1-u_2)^2>\frac{A+1}{A}B, \tag{20}$$

where

$$A=\frac{\rho_2}{\rho_1}\,\frac{\operatorname{cth} m(h+\delta)}{\operatorname{cth} m(a-2\delta)};\quad B=\frac{\sigma m}{\rho_1 \operatorname{cth} m(a-2\delta)}.$$

The value of the wave number m is found as a function of We and γ from Fig. 3.

The piston mode is accompanied by the formation and motion of gas bubbles and liquid connectors whose dimensions agree with the column diameter. The conditions for the origination of such a mode depend on the linear dimensions of the layer and on the discharge characteristics according to (12) and (18). The determination of the stable motion domains of gas or liquid pistons reduces to analyzing the two-dimensional Navier—Stokes equations for two free phase-separation boundaries. This problem is sufficiently complex and can be the topic of a special investigation. Within the framework of the present paper we limit ourselves to approximate estimates which follow from the proposed physical model of dissociation. Experimental results on the velocities of piston motion show that their velocities are practically independent of the gas and liquid volume discharges. This means that the thickness of the fluid piston diminishes with the increase in the gas velocity to a certain critical quantity, which presages the occurrence of dissociation. Since the regularities greatly resemble jet dissociation under the influence of transverse perturbations, the ratio $L^{cr} = 3\pi\sigma/\rho_2 u_4^2$ presented in [7] can then be used to estimate the critical piston thickness. On the other hand, by starting from the discharge characteristics $L^{cr} = \bar{R}(1-\varphi)/\varepsilon\varphi$; hence, piston dissociation is observed in the following case:

$$\frac{1-\varphi}{\varepsilon\varphi} \leqslant \frac{3\pi\sigma}{\rho_2 u_4^2 \bar{R}}. \tag{21}$$

Therefore, the condition for passage from one mode to another is determined successfully as a result of the investigation performed. The problem of the next stage in the research is the experimental verification of the inequalities obtained.

NOTATION

R^{cr}, critical radius; σ, coefficient of surface tension; ρ_1, ρ_2, gas and liquid densities; V, bubble volume; R_0, bubble radius; u_0, velocity of unperturbed bubble motion; φ, gas content; ε, layer porosity; α, geometric dimension of the packing; x, y, coordinates; D, channel diameter; t, time; L, initial spacing between bubbles; ΔL, characteristic packing dimension; $\Delta L = R$; u_1, gas velocity in the computation on the free layer section; m, wave number; ω, frequency; η, deflection of the phase-separation surface; u, v, velocities of unperturbed film motion along the x axis; θ, wettability angle; g, free-fall acceleration; λ, wavelength; u_2, fluid velocity in the computation on a free layer section; u_4, velocity of piston motion, $\bar{R}$, column radius; c, velocity of dynamical waves; μ_2, dynamical viscosity of the fluid; u_{12}, relative velocity of phase motion; p, pressure. Indices: 1, gas phase; 2, fluid; 3, packing; $u_{12}=1-\frac{u_2\varphi}{u_1(1-\varphi)}$; $\mathrm{We}=\frac{\rho_2 u_{12}^2 h}{\sigma}$; $\gamma=\frac{u_{23}}{u_{12}}$; $a=\frac{4\varepsilon R}{3(1-\varepsilon)}$; $D_\lambda=\frac{2\pi h}{\lambda}$.

LITERATURE CITED

1. G. I. Taylor, Proc. Roy. Soc. London, A210, 192 (1960).
2. G. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill (1969).
3. Moises and Griffiths, Teploperedacha, Ser. C, 84, No. 1, 38 (1962).
4. V. I. Sosunov, Yu. I. Petukhov, and N. G. Skorobogatov, Questions of Hydrodynamics and Heat Transfer [in Russian], Nauka, Novosibirsk (1972), p. 28.
5. A. V. Vasil'eva and V. F. Butuzov, Asymptotic Expansion of the Solutions of Singularly Perturbed Equations [in Russian], Nauka, Moscow (1973).
6. Griffiths and Wallace, Teploperedacha, Ser. C, 83, No. 3, 99 (1961).
7. V. A. Borodin, Yu. F. Dityakin, L. A. Klyachko, and V. I. Yagodkin, Atomization of Fluids [in Russian], Mashinostroenie, Moscow (1967).
8. L. M. Milne-Thompson, Theoretical Hydrodynamics, 5th ed., Macmillan (1968).
9. H. Schlichting, Boundary Layer Theory, 6th ed., McGraw-Hill (1968).